Parametrization of the Galactic Structure by two exponentials

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Abstract. We parametrized the total structure of the Galaxy in cylindrical coordinates by radial and vertical exponentials up to $z \sim 10$ kpc, covering thin disc, thick disc, and the inner spheroid. However, we let the scaleheight and scalelength to be a continuous function of distance from the Galactic plane. The standard deviations for the differences between the space densities estimated by means of the newly defined scaleheight and scalelength and the observed space densities for three absolute magnitude intervals, $5 < M(g) \leq 6$, $6 < M(g) \leq 7$, and $7 < M(g) \leq 8$, for the fields SA 114, ELAIS, and #0952+5245 are rather small. The uncertainties for the scaleheight are also small, indicating that this parameter is very sensitive to the distance from the Galactic plane, whereas those for the scalelength are larger.

Key words: Galaxy: structure – Galaxy: fundamental parameters – Galaxy: stellar content

1. Introduction

For some years, a disagreement exists among the researchers about the formation history of our Galaxy. Yet there has been a large improvement about this topic since the pioneering work of Eggen, Lynden-Bell & Sandage (1962) who argued that the Galaxy collapsed in a free-fall time ($\sim 2 \times 10^8$ yr). Now, we know that the Galaxy collapsed over many Gyr (e.g. Yoshii & Saio 1979; Norris, Bessell, & Pickles 1985; Norris 1986; Sandage & Fouts 1987; Carney, Latham, & Laird 1990; Norris & Ryan 1991; Beers & Sommer-Larsen 1995) and at least some of its components are formed from the merger or accretion of numerous fragments, such as dwarf-type galaxies (cf. Searle & Zinn 1978; Freeman & Bland-Hawthorn 2002, and references therein). Also, the number of population components of the Galaxy increased by one, complicating interpretations of any data set. The new component (the thick disc) was introduced by Gilmore & Reid (1983) in order to explain the observation that star counts towards the South Galactic Pole were not in agreement with a single-disc (thin-disc) component, but rather could be much better represented by two such components. This was the simplest combination of free parameters giving a satisfactory fit, and simplicity is generally key in astrophysical fits.

Different parametrization followed the work of Gilmore & Reid (1983). For example, Kuijken & Gilmore (1989) showed that the vertical structure of our Galaxy could be best explained by a multitude of quasi-isothermal components, i.e. a large number of sech^2 isothermal discs, together making up a more sharply peaked sech or exponential distribution.

Also, we quote the work of de Grijs, Peletier & van der Kruit (1997) who made clear that in order to build up a sech distribution, one needs multiple components. Although different parametrization were tried by many researchers, only the one of Gilmore & Wyse (1985) which is based on star counts estimation for thin disc, thick disc and spheroid (halo) became as a common model for our Galaxy, and used widespread with improving the parameters, however. In other words, the canonical density laws are as follows: 1) a parametrization for thin and thick discs in cylindrical coordinates by radial and vertical exponentials and 2) a parametrization for halo by the de Vaucouleurs (1948) spheroid. The thin disc dominates the small z distances from the galactic plane with a scaleheight ranging from 200 to 475 pc (Robin & Crézé 1986), whereas the thick disc extends to larger z distances with larger scaleheight. In some studies, the range of values for the parameters is large, especially for the thick disc. For example, Chen et al. (2001) and Siegel et al. (2002) give 6.5-13 and 6-10 per cent, respectively, for the relative local density for the thick disc. In the paper of Karaali, Bilir & Hamzaoğlu (2004), we discussed the large range of these parameters and claimed that Galactic model parameters are absolute magnitude dependent. We showed that the range of the model parameters estimated for a unique absolute magnitude interval is considerably smaller.

In the present study different procedure is followed. We show that logarithmic space densities can be parametrized by two exponentials for each absolute magnitude interval. However, we let the scaleheight and the scalelength to be a continuous function of the distance from the galactic plane. Also, we show that the efficiency of the thick disc is absolute magnitude dependent.

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In Sections 2 and 3, the canonical density law forms and the new procedure for density evaluation is discussed. The calibration of the scaleheight and scalelength for three absolute magnitude intervals for three fields is given in Section 4. In Section 5 the densities are compared at different distances from the galactic plane evaluated by the new calibration for three fields. Finally Section 6 provides a discussion.

2. The canonical density law forms

Disc structures are usually parameterized in cylindrical coordinates by radial and vertical exponentials,

$$D_i(x,z) = n_i exp(-z/H_i) exp(-(x-R_o)/h_i) \qquad (1)$$

where z is the distance from galactic plane, x is the planar distance from the Galactic center, R_0 is the solar distance to the Galactic center (8.6 kpc), H_i and h_i are the scaleheight and scalelength respectively, and n_i is the normalized local density. The suffix i takes the values 1 and 2, as long as the thin and thick discs are considered. A similar form uses the $sech^2$ (or $sech$) function to parametrize the vertical distribution for the thin disc,

$$D_1(x,z) = n_1 sech^2(-z/H_1^{'}) exp(-(x-R_o)/h_1). \qquad (2)$$

Because the $sech$ function is the sum of two exponentials, $H_1^{'}$ is not really a scaleheight, but has to be compared to H_1 by dividing it with 2: $H_1 = H_1^{'}/2$ (van der Kruit & Searle 1981a, b, 1982a, b; van der Kruit 1988). However, in order to build up a $sech$ distribution, one needs multiple components (Kuijken & Gilmore 1989, de Grijs et al. 1997).

The density law for the spheroid (halo) component is parameterized in different forms. The most common is the de Vaucouleurs (1948) spheroid used to describe the surface brightness profile of elliptical galaxies. This law has been deprojected into three dimensions by Young (1976) as

$$D_s(R) = n_s \ exp[-7.669(R/R_e)^{1/4}]/(R/R_e)^{7/8}, \qquad (3)$$

where R is the (uncorrected) Galactocentric distance in spherical coordinates, R_e is the effective radius and n_s is the normalized local density. R has to be corrected for the axial ratio $\kappa = c/a$,

$$R = [x^2 + (z/\kappa)^2]^{1/2}, \qquad (4)$$

where,

$$x = [R_o^2 + (z/\tan b)^2 - 2R_o(z/\tan b)\cos l]^{1/2}, \qquad (5)$$

b and l being the Galactic latitude and longitude, respectively, for the field under investigation.

An alternative formulation is the power law,

$$D_s(R) = n_s/(a_o^n + R^n) \qquad (6)$$

where a_o is the core radius.

If one restricts the work to vertical direction, then the third factor in eqs. (1) and (2) can be neglected.

3. Unique equation for space densities

We mentioned in Section 1 that the introduction of the thick disc component into the literature was due to better representation of the observed star counts in the South Galactic Pole. Principally, what we are doing is matching the observed data with the appropriate estimated data. Hence, we can modify and limit the density law forms cited in Section 2 for this purpose. We can argue that the total structure of our Galaxy can be parametrized in cylindrical coordinates by radial and vertical exponentials in the form of eq. (1) up to many kiloparsecs, covering thin and thick discs and probably the inner spheroid. However, we let the scaleheight and the scalelength to be a continuous function of distance from the galactic plane. Our claim is based on the following:

1. The density law forms for thin and thick discs are similar. There may be a continuous transition for the scaleheight from short distances to large distances in our Galaxy. In the canonical way, we estimate two scaleheights for two discs which are the mean values for relatively short and large distances. The same case is valid for the scalelength.
2. Although there is a tendency in the recent works (cf. Feltzing, Bensby, & Lundström, 2003) that the two discs are discrete components, their kinematical data and $[Fe/H]$ metallicities overlap.
3. There is almost a consensus for the double structure of the spheroid, inner and outermost spheroids. According to Norris (1996), there are a number of indications that a significant fraction of material with $[Fe/H] < -1$ has disclike signature. Hence, the density law form for inner spheroid may be similar to the density law form of discs.

The most important point at this approach is the calibration of the scaleheight H and scalelength h with the distance z from the galactic plane. We followed the following procedure for this purpose. First we write eq. (1) in the logarithmic form:

$$D^*(x,z) = n^* - (\frac{z}{H} + \frac{x-R_o}{h})\log(e) \qquad (7)$$

where $D^*(x,z) = \log D(x,z) + 10$ and $n^* = n + 10$. If x of (5) is substituted in eq. (7) and re-written, it takes the form

$$n^* - D^*(z,l,b) = \{(z/H) + (1/h)[R_0^2 + (z/\tan b)^2$$
$$-2R_o(z/\tan b)\cos l]^{1/2} - R_0]\}\log(e) \qquad (8)$$

Now, we can use the Hipparcos' local space density (Jahreiss & Wielen, 1997) for a specific absolute magnitude interval $M_1 - M_2$ and a sequence of observed $D^*(z,l,b)$ space densities and estimate the most likely H and h values, for each element of this sequence.

4. Absolute magnitude dependent scaleheight and scalelength as a function of distance from the galactic plane

The eq. (8) applied to the data of three fields whose galactic coordinates and sizes are given in Table 1. Fields SA
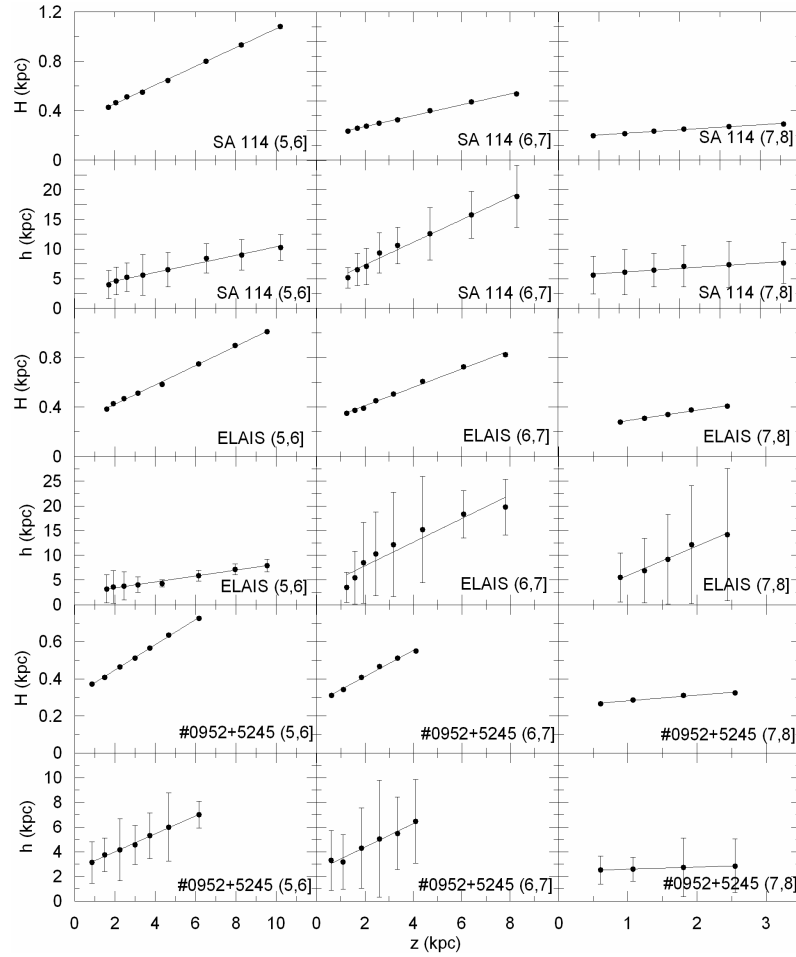
Fig. 1. Calibration of the scaleheight and scalelength with distance from the galactic plane for the absolute magnitude interval (5,6], (6,7], and (7,8], for the fields SA 114, ELAIS, and #0952+5245. The errors bars for the scaleheight could not be demonstrated, since they are small (see Table 2).

Table 1. Galactic coordinates (epoch 2000) and the sizes for three fields investigated.

Field	l	b	Size (deg^2)
SA 114	$68^o.50$	$-48^o.38$	4.239
ELAIS	$84^o.27$	$+44^o.90$	6.571
#0952+5245	$83^o.38$	$+48^o.55$	20

114 and ELAIS are almost symmetric relative to the galactic plane, and the galactic coordinates of the centre of ELAIS and #0952+5245 almost coincide with one another, however their sizes are quite different.

For the field SA 114, we transfered the mean z^* distances from the galactic plane and the corresponding logarithmic space densities D^* for the absolute magnitude intervals $5 < M(g^{'}) \leq 6$, $6 < M(g^{'}) \leq 7$, and $7 < M(g^{'}) \leq 8$ from the work of Karaali et al. (2004), whereas the z^* and D^* data for the same absolute magnitude intervals, for the fields ELAIS and #0952+5245 were recently evaluated by Bilir, Karaali & Gilmore (2005) and Karaali et al. (2005), respectively. The space densities for these intervals extend up

to $\sim$10 kpc from the galactic plane and cover thin and thick discs, and spheroid. Here, $D^* = \log D + 10$, $D = N/\Delta V_{1,2}$; $\Delta V_{1,2} = (\pi/180)^2(\square/3)(r_2^3 - r_1^3)$; $\square$ denotes the size of the field; r_1 and r_2 denote the limiting distance of the volume $\Delta V_{1,2}$; and N denotes the number of stars in this volume. The sizes of the fields SA114, ELAIS, and #0952+5254 are 4.239, 6.751, and 20 deg^2 respectively (see also Table 1). The most likely scaleheights and scalelengths for these intervals are given in Table 2, and the calibrations of scaleheights and scalelengths in this table with the distance from the galactic plane are shown in Fig. 1. We used the procedure of Phleps et al. (2000) for the error estimation in Table 2 and Fig. 1, i.e. changing the values of the parameters until χ^2 increases or decreases by 1. The errors for the scaleheight are rather small, indicating that this parameter is a strong function of the distance from the galactic plane. Whereas, the errors for the scalelenght are large which show that the scalelenght is not effective in the comparison of observed logarithmic space density and the estimated one in eq. (7). The correlation is higher for the calibration of scaleheight for all absolute magnitudes and for all fields. We adopted the following linear

Table 2. The most likely scaleheight (H) and scalelength (h) values for a given distance from the galactic plane (z^*) and the corresponding logarithmic space density D^*, for the absolute magnitude intervals (5,6], (6,7], and (7,8], for the fields SA 114, ELAIS, and #0952+5245. The local space density of Hipparcos are $n^* = 7.47$, for absolute magnitude intervals (5,6], (6,7] and $n^* = 7.48$, for (7,8].

$M(g) \rightarrow$	(5,6]				(6,7]				(7,8]			
Field	z^* (kpc)	D^*	H (kpc)	h (kpc)	z^* (kpc)	D^*	H (kpc)	h (kpc)	z^* (kpc)	D^*	H (kpc)	h (kpc)
SA 114	1.69	5.68	0.43±0.007	4.00±2.38	1.29	6.00	0.39±0.012	5.15±1.76	0.93	6.23	0.33±0.009	5.60±3.15
	2.06	5.46	0.46 0.013	4.61 2.34	1.68	5.73	0.43 0.009	6.55 2.71	1.32	5.83	0.36 0.009	6.10 3.82
	2.60	5.17	0.51 0.007	5.24 2.43	2.06	5.47	0.46 0.008	7.05 3.07	1.68	5.57	0.39 0.005	6.44 2.83
	3.38	4.69	0.55 0.009	5.60 3.43	2.59	5.17	0.50 0.006	9.35 3.39	2.05	5.31	0.42 0.008	7.10 3.49
	4.63	4.21	0.65 0.004	6.51 2.90	3.35	4.73	0.54 0.011	10.59 3.09	2.61	4.93	0.46 0.004	7.39 3.89
	6.53	3.75	0.80 0.009	8.45 2.48	4.68	4.35	0.67 0.006	12.57 4.41	3.29	4.47	0.49 0.006	7.65 3.44
	8.28	3.39	0.93 0.012	9.01 2.60	6.40	3.86	0.79 0.008	15.74 3.95				
	10.21	3.11	1.08 0.012	10.26 2.19	8.27	3.36	0.90 0.007	18.85 5.22				
ELAIS	1.60	5.66	0.39±0.002	3.20±2.83	1.24	5.94	0.35±0.007	3.50±3.06	0.89	6.10	0.28±0.006	5.50±4.94
	1.93	5.51	0.43 0.005	3.60 3.31	1.58	5.63	0.37 0.010	5.46 5.35	1.24	5.75	0.31 0.007	6.90 6.51
	2.46	5.17	0.47 0.003	3.80 2.85	1.94	5.31	0.39 0.006	8.50 8.20	1.58	5.46	0.34 0.006	9.20 9.08
	3.15	4.77	0.51 0.008	4.04 1.57	2.45	5.11	0.45 0.006	10.30 8.46	1.92	5.26	0.38 0.003	12.15 11.95
	4.35	4.17	0.58 0.009	4.32 0.73	3.18	4.73	0.51 0.003	12.20 10.55	2.44	4.87	0.41 0.003	14.20 13.40
	6.15	3.80	0.75 0.007	5.90 1.07	4.38	4.32	0.61 0.002	15.20 10.76				
	7.97	3.47	0.90 0.008	7.17 1.12	6.07	3.80	0.72 0.013	18.30 4.80				
	9.54	3.17	1.01 0.009	7.93 1.27	7.81	3.30	0.82 0.014	19.76 5.64				
#0952+5245	0.86	6.47	0.37±0.004	3.14±1.68	0.61	6.62	0.31±0.006	3.30±2.42	0.60	6.50	0.27±0.005	2.52±1.15
	1.48	5.90	0.41 0.006	3.75 1.36	1.10	6.08	0.34 0.004	3.17 2.20	1.08	5.86	0.29 0.005	2.60 0.97
	2.25	5.36	0.46 0.006	4.16 2.51	1.86	5.50	0.41 0.002	4.30 3.25	1.80	4.97	0.31 0.004	2.74 2.36
	2.99	4.93	0.51 0.004	4.55 1.58	2.59	5.05	0.47 0.002	5.05 4.73	2.55	4.07	0.32 0.005	2.86 2.17
	3.73	4.59	0.57 0.004	5.30 1.85	3.34	4.62	0.51 0.005	5.50 2.93				
	4.66	4.26	0.64 0.005	6.00 2.77	4.10	4.21	0.55 0.005	6.46 3.38				
	6.17	3.72	0.73 0.006	7.01 1.08								

equations in the calibrations and this resulted in the fluctuations appearing on the figures to be smoothed.

$$H = a_1 z + a_0 \qquad (9)$$

$$h = b_1 z + b_0 \qquad (10)$$

The coefficients for the eqs. (9) and (10), evaluated by least-square method, are given in Table 3 together with their rather small uncertainties. All the scaleheights and scalelengths are increasing functions of the distance from the galactic plane. Hence, the scaleheight and scalelength increases from short z distances to the large ones. The numerical values of the scaleheight for short z distances are in the range of the scaleheight for thin disc claimed so far. For example, the scaleheight for the absolute magnitude interval $7 < M(g) \le 8$ for the field #0952+5245 lies within 0.27 and 0.32 kpc. However, the upper limits for the scaleheights for the same absolute magnitude interval, for the fields SA 114 and ELAIS are larger, i.e. 0.49 and 0.41 kpc, respectively. For larger z distances, the numerical value of scaleheight is close to the scaleheight of thick disc appeared in the literature. For example, $H \sim 0.5$ kpc at the distance from the galactic plane $z \sim 3.5$ kpc, for three fields. The scaleheight extends up to $H \sim 1$ kpc at $z \sim 10$ kpc. The range of the scalelength is rather large, and it differs from field to field. The least range as well as the least numerical values belong to the absolute magnitude interval $7 < M(g) \le 8$ for #0952+5245, $2.5 < h < 2.9$ kpc, and the largest ones belong to $6 < M(g) \le 7$ for ELAIS field, $3.5 < h < 19.8$ kpc. Such large scalelengths have not been claimed in the literature up to now.

Table 3. Coefficients for eqs. (9) and (10) for three absolute magnitude intervals for the three fields.

$M(g)$	a_1	a_0	b_1	b_0
SA 114				
(5,6]	0.0760±0.0007	0.3020±0.0041	0.7210±0.0385	3.1607±0.2205
(6,7]	0.0734 0.0020	0.3068 0.0090	1.9057 0.0953	3.5076 0.4235
(7,8]	0.0688 0.0040	0.2704 0.0085	0.8906 0.1166	4.9499 0.2487
ELAIS				
(5,6]	0.0784±0.0018	0.2654±0.0094	0.5976±0.0310	2.2205±0.1677
(6,7]	0.0744 0.0026	0.2621 0.0110	2.3905 0.2977	3.0923 1.2495
(7,8]	0.0837 0.0054	0.2073 0.0092	5.9479 0.4658	-0.0100 0.7923
#0952+5245				
(5,6]	0.0683±0.0013	0.3108± 0.0047	0.7261±0.0686	2.5468±0.2468
(6,7]	0.0707 0.0038	0.2713 0.0099	0.9526 0.0803	2.4696 0.2066
(7,8]	0.0303 0.0041	0.2515 0.0068	0.1769 0.0050	2.4134 0.0083

Table 4. Standard deviations for the differences between the observed and evaluated logarithmic space densities for three absolute magnitude intervals, for three fields.

Field	SA 114	ELAIS	#0952+5245
$M(g)$	s	s	s
(5,6]	± 0.03	± 0.06	± 0.02
(6,7]	± 0.05	± 0.07	± 0.04
(7,8]	± 0.04	± 0.03	± 0.03

5. Testing the new calibrations

We replaced the calibrations of scaleheights and scalelengths into eq. (8), and we evaluated the logarithmic space densities, D^*, for the z^* distances for which observed space densities are available in Table 2 for three fields. Then, we compared them with the observed space densities. The standard deviations for differences between the observed and evaluated space densities are small (Table 4), confirming the new calibration, i.e. the total structure of the Galaxy can be

Table 5. Comparison of the logarithmic space densities (D^*) evaluated for a sequence of distance from the galactic plane (z), for three absolute magnitude intervals, for three fields.

$M(g) \rightarrow$	(5,6]			(6,7]				(7,8]		
z (kpc)	SA 114	ELAIS	#0952+5245	SA 114	ELAIS	#0952+5245	z (kpc)	SA 114	ELAIS	#0952+5245
0	7.47	7.47	7.47	7.47	7.47	7.47	0	7.48	7.48	7.48
1	6.28	6.21	6.33	6.30	6.18	6.21	1	6.17	5.99	5.95
2	5.48	5.41	5.53	5.51	5.35	5.37	2	5.30	5.16	4.70
4	4.47	4.42	4.47	4.50	4.35	4.31	3	4.67	4.63	3.66
6	3.85	3.82	3.79	3.89	3.76					
8	3.44	3.42		3.48	3.37					
10	3.14	3.12								

parametrized in cylindrical coordinates by radial and vertical exponentials up to many kiloparsecs (~ 10 kpc), covering the thin disc, the thick disc, and the halo. However, contrary to the procedures in situ, the scaleheight and scalelength are not constants, but they are continuous functions of distance from the galactic plane.

We have a parametrization for each absolute magnitude interval, for each field. Now, a question: Do the parametrizations for a specific absolute magnitude interval for three fields produce the same space density for a given distance from the galactic plane? The answer to this question can be obtained from Table 5 where logarithmic space densities are given for three absolute magnitude intervals for three fields. For the absolute magnitude interval $5 < M(g) \leq 6$, the field #0952+5245 is investigated only up to $z \sim 6$ kpc, whereas the space densities for the fields SA 114, and ELAIS extend up to $z \sim 10$ kpc. The logarithmic space densities for three fields are rather close to each other for a given z. For the absolute magnitude interval $6 < M(g) \leq 7$, the agreement of the logarithmic space densities is also good for the fields #0952+5245 and ELAIS, however it is a bit less for SA 114. The logarithmic space densities for $7 < M(g) \leq 8$ extends up to only $z \sim 3$ kpc, and it is interesting, that the agreement is better between the data of SA 114 and ELAIS fields.

6. Discussion

The parametrization of the Galactic components has a long history. Bahcall & Soneira (1980) parametrized the disc in cylindrical coordinates by radial and vertical exponentials, and the halo by a de Vaucouleurs (1948) spheroid in their two component Galactic model. Whereas, Gilmore & Reid (1983) introduced a new component, thick disc, in order to explain their star count observations. Later, it was noticed that the third component was a rediscovery of Intermediate Population II which was named in the Vatican conference (O'Connell, 1958).

Different parametrizations followed the work of Gilmore & Reid (1983). For example, Kuijken & Gilmore (1989) showed that the vertical structure of our Galaxy could be best explained by a multitude of quasi-isothermal components, i.e. a large number of $sech^2$ isothermal discs, together making up a more sharply peaked sech or exponential distribution. Also, we quote the work of de Grijs et al. (1997) who made clear that in order to build up a sech distribution, one needs multi-

ple components. Although various parametrization were tried by many researchers, only the one of Gilmore & Wyse (1985) became as a common model for our Galaxy. In other words, the canonical density laws are as follows: 1) a parametrization for the thin disc, 2) a parametrization for the thick disc both in cylindrical coordinates by radial and vertical exponentials, and 3) a parametrization for the halo by the de Vaucouleurs (1948) spheroid.

In some studies, the range of values for the parameters is large, especially for the thick disc. For example, Chen et al. (2001) and Siegel et al. (2002) give 6.5-13 and 6-10 per cent, respectively, for the relative local density for the thick disc. In the paper of Karaali et al. (2004), we discussed the large range of these parameters and claimed that Galactic model parameters are absolute magnitude dependent. We showed that the range of the model parameters estimated for a unique absolute magnitude interval is considerably smaller.

In this paper, we used the results obtained in our previous paper, thus we estimated model parameters as a function of absolute magnitude. Additionally, we showed that the derived logarithmic space densities could be parametrized by two exponentials only, for each absolute magnitude interval, without regarding the population type of stars. However, we let the scaleheight and scalelength to be a continuous function of distance from the galactic plane. This is the main difference between the works of Kuijken & Gilmore (1989), de Grijs et al. (1997) and our work. The starting point is that the structure of two discs are parametrized in the same way, i.e. by two exponentials, and that there are a number of indications that a significant fraction of material with $[Fe/H] < -1$ (halo material) has disclike signature (Norris 1996). Here we showed that the scaleheight and the scalelength could be calibrated with z distance from the galactic plane by linear functions, and the space densities evaluated for the absolute magnitude intervals $5 < M(g) \leq 6$, $6 < M(g) \leq 7$, and $7 < M(g) \leq 8$ for three fields could be explained by two exponentials with these scaleheights and scalelengths. The (small) standard deviations corresponding to the differences between the observed logarithmic space densities and the evaluated ones by the calibrations (Table 4) strongly confirm our suggestion. The scaleheights corresponding to small z distances are typical thin disc scaleheights, whereas those at larger z distances are at the level of thick disc scaleheights (Table 2). For example, the scaleheight for stars with $7 < M(g) \leq 8$ at $z = 0.6$ kpc for the field #0952+5245 is $H = 0.27$ kpc, and the one for stars with $5 < M(g) \leq 6$ at $z = 4.66$ kpc is $H = 0.64$

Table 6. Galactic model parameters estimated by the data of stars with $5 < M(g) \leq 10$ and $4 < M(g) \leq 9$, respectively, in fields ELAIS and #0952+5245, by the procedure in situ. Symbols: n^* is the logarithmic local space density, H is the scaleheight, n/n_1 is the local space density relative to the thin disc and κ is the axial ratio for the halo.

Field	Parameter	Thin disc	Thick disc	Halo
ELAIS	n^*	$7.51^{+0.03}_{-0.02}$	$6.26^{+0.07}_{-0.07}$	$4.48^{+0.37}_{-0.53}$
	H (pc)	274^{+8}_{-6}	807^{+78}_{-61}	–
	n/n_1 (per cent)	–	5.62	0.09
	κ	–	–	$0.65^{+0.35}_{-0.28}$
#0952+5245	n^*	$7.43^{+0.04}_{-0.05}$	$6.14^{+0.09}_{-0.11}$	$4.16^{+0.22}_{-0.41}$
	H (pc)	276^{+12}_{-12}	895^{+114}_{-96}	–
	n/n_1 (per cent)	–	5.13	0.05
	κ	–	–	$0.59^{+0.31}_{-0.21}$

kpc. In Table 5, we compare the logarithmic space densities (D^*) evaluated by the new calibration for a sequence of distance from the Galactic plane (z), for three absolute magnitude intervals, for three fields. For the absolute magnitude interval $5 < M(g) \leq 6$, the D^*-values for a given z are rather close to each other for three fields. The same argument holds for the interval $6 < M(g) \leq 7$, however the agreement between the data of ELAIS field and #0952+5245 is better, and finally for $7 < M(g) \leq 8$ the agreement between the data of SA 114 and ELAIS favors. These small differences are real and they reflect the differences between the corresponding coefficients in Table 3. Actually, the numerical values for a_1 and a_0 for the absolute magnitude interval $5 < M(g) \leq 6$ for three fields are rather close to each other, whereas for the interval $6 < M(g) \leq 7$, a_0 is 0.26 and 0.27 for the fields ELAIS and #0952+5245 respectively, but it is 0.31 for the field SA 114. From the other hand, for the interval $7 < M(g) \leq 8$, a_1 is 0.07 and 0.08 for the fields SA 114 and ELAIS, but it is 0.03 for the field #0952+5245. We evaluated the number of stars per deg^2 for the absolute magnitude intervals $6 < M(g) \leq 7$ and $7 < M(g) \leq 8$ for three fields to find out its effect on the agreement between the space densities for a specific z distance in question. We found that there is no any correlation between the number of stars per deg^2 and the space densities for any of the absolute magnitude interval claimed above, for three fields. The claimed small differences is not an unexpected case. It is worthwhile to emphasize that the Galactic model parameters determined, by means of the data in different directions of the Galaxy, in situ do not overlap either. Table 1 of Karaali et al. (2004) which gives the Galactic model parameters for recent works confirms our claim. We can give an additional example. Although the Galactic coordinates of the ELAIS field and #0952+5245 are almost the same, the corresponding model parameters determined in situ for these fields are not the same (Table 6). Probably, the difference originates from their sizes (see Table 1) and/or from different absolute magnitude intervals by which the model parameters were estimated, i.e. $5 < M(g) \leq 10$ and $4 < M(g) \leq 9$ for the ELAIS field and for the field #0952+5245, respectively.

It is worthwhile to note that small-number statistics could affect the robustness of the results presented. One can notice immediately that the uncertainty for such statistics is large. For example, the standard deviation or its equivalent value, the probable error, is large due to the small number in the denominator of the expression defining it.

We can generalize this topic as such: In many areas of astronomy and astrophysics it occasionally happens that only small number of events of interest are detected during an observation. Examples range from the number of supernovae seen in a given period of time from a cluster of galaxies to the number of gamma rays detected during a source observation. If the goal is to determine quantities such as the event rate or the ratio of different event types, then the best approach would be to repeat the measurement with a longer integration time or a larger collection factor in order to obtain enough events for an accurate measurement. In some cases, for one reason or another, this is not possible or practical, and one is forced to make the best use of the data in hand. Results are then typically quoted as upper limits at a specified confidence level or as a measured value with error bars containing a specified confidence interval.

Here we pose a second question: Does this work refuse the multistructure of the Galaxy? The answer could be "yes" if this question was asked about fifteen years ago. Today, there is a consensus about the existence of the thick disc not only in our Galaxy but also in external galaxies. However, there are some points that remains to be explained. We direct the reader to the work of Feltzing et al. (2003) and the references therein. According to these authors, the ages of thin disc (6.1 ± 3.8 Gyr) and thick disc (12.1 ± 2.0 Gyr) do not overlap, indicating two discrete components for our Galaxy. However, the $[Fe/H]$ metal abundances for two disc overlap in the range $-0.8 < [Fe/H] < -0.3$ (Nissen, 2003). Although the trend of α-elements is different for two discs for $[Fe/H] < 0$, giving a chance to separate thin and thick disc stars, this is not the case for $[Fe/H] \geq 0$. According to Feltzing et al. (2003) and Bensby et al. (2004a, b), metal rich stars abundant in α-elements can be explained as the long formation time of thick disc. There is overlapping also for kinematical data, as claimed by many authors. The "transition objects" claimed recently by Bensby et al. (2004b) is a good confirmation of this argument.

The points claimed in the previous paragraph indicate the relation between thin and thick discs. Different data of these components overlap. They are not isolated parts of our Galaxy, but complementary ones. Hence, we may adopt the same argument for the distribution of their space density, as we carried out in this work.

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